

2025-DSE
MATH EP
M2

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2025

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

8:30 am – 11:00 am (2½ hours)

This paper must be answered in English

INSTRUCTIONS

- (1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9, 11 and 13.
- (2) This paper consists of TWO sections, A and B.
- (3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
- (4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.
- (5) Unless otherwise specified, all working must be clearly shown.
- (6) Unless otherwise specified, numerical answers must be exact.
- (7) The diagrams in this paper are not necessarily drawn to scale.
- (8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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Candidate Number



$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$ $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$	$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$ $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$ $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$ $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$
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SECTION A (50 marks)

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2. Let $f(x) = \frac{2x}{\tan x}$, where $0 < x < \frac{\pi}{2}$.

(a) Prove that $f\left(\frac{\pi}{4} + h\right) - f\left(\frac{\pi}{4}\right) = \frac{2h - 2h \tan h - \pi \tan h}{1 + \tan h}$.

(b) Find $f'\left(\frac{\pi}{4}\right)$ from first principles.

(5 marks)

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3. (a) Find the constant k such that $\frac{d}{dx} \ln(\sqrt{x^4 + 9} - x^2) = \frac{kx}{\sqrt{x^4 + 9}}$.

- (b) At any point (x, y) on the curve Γ , the slope of the tangent to Γ is $\frac{4x^3 - x}{\sqrt{x^4 + 9}}$. It is given that Γ passes through the point $(2, 5)$. Does Γ pass through the point $(-2, 5)$? Explain your answer.

(6 marks)

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4. Let $C = \begin{pmatrix} k & 1 \\ -1 & 2 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & k-3 \\ k+3 & -4 \end{pmatrix}$, where k is a real number. Find k such that $CD = DC$.

(3 marks)

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- (b) Using (a), evaluate $\sum_{k=10}^{60} (-1)^k k^2 (k^2 - 1)$.

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6. Let $\frac{\pi}{2} < x < \frac{3\pi}{4}$.

(a) Find a pair of constants h and k such that $\frac{\sec x + \tan x}{\csc x - \cot x} + \frac{\sec x - \tan x}{\csc x + \cot x} \equiv h \sec x \csc x + k$.

(b) Solve the equation $\frac{\sec x + \tan x}{\csc x - \cot x} + \frac{\sec x - \tan x}{\csc x + \cot x} + 6 = 0$.

(5 marks)

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8. Consider the parabola $C : y = x^2 + 4$. C cuts the y -axis at the point P . Q is a moving point on C with h as its x -coordinate, where $h > 0$. The vertical line which passes through Q cuts the x -axis at the point R . Denote the origin by O . Let A be the area of the quadrilateral $OPQR$.

- (a) By expressing A in terms of h , find $\frac{dA}{dh}$.
- (b) Suppose that $h = 3 - 2e^{-t}$, where t ($t > 0$) is the time measured in seconds. Someone claims that the rate of change of A has only one maximum value. Do you agree? Explain your answer. (7 marks)

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10. Define $f(x) = \frac{x^3 + 25x}{x^2 + 3}$ for all $x \in \mathbb{R}$. Denote the graph of $y = f(x)$ by G .

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11. (a) Let m be a positive integer. Express $\int_0^\pi e^{-x} \sin 2mx \, dx$ in terms of m . (3 marks)

(b) (i) Evaluate $\int_0^\pi e^{-x} \sin 5x \cos 3x \, dx$.

(ii) Evaluate $\int_0^{2\pi} e^{-x} \sin 5x \cos 3x \, dx$. (6 marks)

(c) Evaluate $\int_{e^{-\frac{3\pi}{2}}}^{e^{\frac{\pi}{2}}} \cos(\ln x^5) \sin(\ln x^3) \, dx$. (4 marks)

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12. Consider the system of linear equations in real variables x, y, z

$$(E): \begin{cases} x - (a+1)y + 2z = 1 \\ 2x + ay + 7z = 2 \\ 4x + (a+6)y + (3a+14)z = b \end{cases}, \text{ where } a, b \in \mathbf{R}.$$

(a) Assume that (E) has a unique solution.

(i) Find the range of values of a .

(ii) Express y in terms of a and b .

(5 marks)

(b) Assume that (E) has infinitely many solutions and $a > 0$.

(i) Solve (E) .

(ii) Let k be a real constant. If all the solutions of (E) satisfy $kx + y + 4kz^2 > 0$, find the range of values of k .

(7 marks)

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13. The position vectors of the points A , B and C are $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $-2\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{k}$ respectively. Denote the plane containing A , B and C by Π . Let E be the projection of the point $D(-2, -6, 2)$ on Π . It is given that EA produced meets BC at the point F .

(a) Find $\vec{AB} \times \vec{AC}$. (3 marks)

(b) Find $\vec{ED}$ and $\vec{EA}$. (4 marks)

(c) If $BF : FC = 1 : t$, find t . (3 marks)

(d) Find the volume of the pyramid $ABDF$. (3 marks)

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END OF PAPER

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