

**MATHEMATICS Extended Part**  
**Module 2 (Algebra and Calculus)**  
**Question-Answer Book**

8:30 am – 11:00 am (2½ hours)  
This paper must be answered in English

**INSTRUCTIONS**

- (1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9, 11 and 13.
- (2) This paper consists of TWO sections, A and B.
- (3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
- (4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.
- (5) Unless otherwise specified, all working must be clearly shown.
- (6) Unless otherwise specified, numerical answers must be exact.
- (7) The diagrams in this paper are not necessarily drawn to scale.
- (8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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Candidate Number



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- (3 marks)

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2. Let  $x > 0$ .

(a) Find  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ .

(b) Find  $\frac{d}{dx} e^{\sqrt{x}}$  from first principles.

(5 marks)

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3. Let  $m$  be a positive integer. Consider the expansion of  $\left(x^m - \frac{2}{x}\right)^{24}$  in descending powers of  $x$ .

(a) Find the first 3 terms of the expansion.

(b) If the 19th term of the expansion is a constant, find  $m$  and the coefficient of  $x^{60}$  in the expansion. (5 marks)

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4. (a) Let  $0 < x < \frac{\pi}{2}$ . Prove that  $\csc 2x - \cot 2x = \tan x$ .

(b) Solve the equation  $(\csc 3\theta - \cot 3\theta)(\csc \theta - \cot \theta) = 1$ , where  $\frac{\pi}{6} < \theta < \frac{\pi}{3}$ .

(5 marks)

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5. (a) Let  $k$  be a constant.

Using integration by parts, prove that  $\int \cos(k \ln x) dx = \frac{x}{1+k^2} (\cos(k \ln x) + k \sin(k \ln x)) + \text{constant}.$

- (b) Using (a), or otherwise, evaluate  $\int_1^e \sin^2(\pi \ln x) dx.$

(6 marks)

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6. Consider the system of linear equations in real variables  $x, y, z$

$$(E): \begin{cases} 3x + y - 9z = 0 \\ 2x + y - 7z = 0 \end{cases}$$

(a) Solve  $(E)$ .

(b) Someone claims that  $(E)$  has a unique solution  $(x, y, z)$  satisfying  $\sin x + \cos y - \cos z = 0$ , where  $0 < z < \frac{\pi}{2}$ . Do you agree? Explain your answer.

(6 marks)

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7. Let  $C$  be the curve  $x^2y + 2xy^2 + 8 = 0$ .

(a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

(b) Someone claims that two of the tangents to  $C$  are parallel to the straight line  $x + 2y + 1 = 0$ . Do you agree? Explain your answer.

(6 marks)

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8. (a) Using mathematical induction, prove that  $\sum_{r=1}^n r(2^{-r}) = 2 - (n+2)(2^{-n})$  for all positive integers  $n$ .

(b) Evaluate

(i)  $\sum_{r=1000}^{1999} r(2^{-r})$ ,

(ii)  $\sum_{r=1}^{1000} (2000 - r)2^r$ .

(7 marks)

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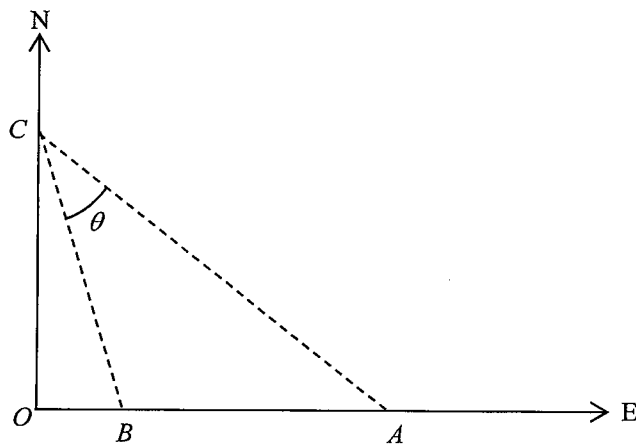
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- (7 marks)

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11. (a) Let  $a$  be a non-zero constant. Using integration by substitution, find  $\int \frac{1}{x^2 + a^2} dx$  in terms of  $a$ . (3 marks)

(b) Let  $g(x)$  and  $h(x)$  be continuous functions defined on  $\mathbf{R}$ . It is given that  $g(x)$  is an even function and  $h(x)$  is an odd function. Prove that  $\int_{-c}^c \frac{g(x)}{1 + e^{h(x)}} dx = \int_0^c g(x) dx$ , where  $c$  is a constant. (4 marks)

(c) Evaluate  $\int_{-1}^1 \frac{3^x + 3^{-x}}{(1 + e^{\sin^3 x})(9^x + 9^{-x} + 7)} dx$ . (5 marks)

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12. (a) Consider the system of linear equations in real variables  $x, y, z$

$$(E): \begin{cases} 2x & + z = -2 \\ -(2\lambda+5)x + \lambda y & = -1 \\ (\lambda+2)x + y - z & = 0 \end{cases}, \text{ where } \lambda \in \mathbf{R}.$$

Assume that  $(E)$  has a unique solution.

- (i) Find the range of values of  $\lambda$ .

- (ii) Solve  $(E)$ .

(6 marks)

- (b) Let  $\mathbf{u} = h\mathbf{i} + (2h+5)\mathbf{j} - 2h\mathbf{k}$  for any  $h > 0$ . Suppose that the vector  $\mathbf{v}$  is perpendicular to  $(h+2)\mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\mathbf{u} \times \mathbf{v} = \mu\mathbf{i} + 2h\mathbf{j} - \mathbf{k}$ , where  $\mu$  is a real number.

- (i) Express  $\mathbf{v}$  and  $\mu$  in terms of  $h$ .

- (ii) Does there exist a value of  $h$  such that the area of the parallelogram with adjacent sides  $\mathbf{u}$  and  $\mathbf{v}$  is 9? Explain your answer.

(7 marks)

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13. Let  $A = \begin{pmatrix} -2 & 2 \\ 1 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 4 & 2 \\ 1 & 5 \end{pmatrix}$ .

(a) Evaluate  $A - 6A^{-1}$  and  $ABA^{-1}$ . (4 marks)

(b) Find  $B^n$ , where  $n$  is a positive integer. (3 marks)

(c) Does there exist a positive integer  $k$  such that  $A^{2k}B^{2k}(A^{-1})^{2k} = (ABA^{-1})^{2k}$ ? Explain your answer. (3 marks)

(d) Evaluate  $A^{999}B^{999}(A^{-1})^{999}$ . (2 marks)

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