

2024-DSE
MATH EP
M1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2024

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8:30 am – 11:00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

- (1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.
- (2) This paper consists of TWO sections, A and B.
- (3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
- (4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.
- (5) Unless otherwise specified, all working must be clearly shown.
- (6) Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.
- (7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

©香港考試及評核局 保留版權
Hong Kong Examinations and Assessment Authority
All Rights Reserved 2024

Please stick the barcode label here.

Candidate Number



SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X , where a and b are constants such that $6 < b < 15$.

x	0	3	6	b	15
$P(X=x)$	0.3	a	0.1	0.2	0.2

It is given that $\text{Var}(5X) = 739$.

- (a) Find a and b .
- (b) Let C be the event that $0 < X \leq 7$.
- (i) Let D be the event that $4 < X \leq 15$. Are C and D independent? Explain your answer.
- (ii) Let E be an event such that $P(E) \neq 0$. If C and E are mutually exclusive, write down the greatest possible value of $P(E)$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

2. In an orchestra, $\frac{3}{5}$ of the members wear glasses. Among the male members, $\frac{4}{9}$ of them wear glasses. The probability that a randomly selected member is a female not wearing glasses is $\frac{3}{20}$.

(a) Given that a randomly selected member does not wear glasses, find the probability that the member is a female.

(b) Find the probability that a randomly selected member is a female wearing glasses. (6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

3. When a coin is tossed, the probability of getting a tail is p , where $0 < p < 1$. When the coin is tossed 20 times, the ratio of the probability of getting 1 tail to the probability of getting 3 tails is 49 : 57.

(a) Find p .

(b) The coin is tossed k times. Find the least value of k so that the probability of getting at least 1 tail is greater than 0.85.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

4. The weekly revision time of each student in a school follows a normal distribution with a mean of μ hours. A random sample of 81 students is drawn from the school. The mean and the standard deviation of the weekly revision time of these students are 13 hours and 1.75 hours respectively.

- (a) The width of a $\beta\%$ confidence interval for μ is 0.7. Find β .
- (b) It is given that there are 36 boys in the sample, and the mean and the standard deviation of the weekly revision time of these boys are 12.5 hours and 2 hours respectively. Find the standard deviation of the weekly revision time of the girls in the sample.

[Hint: The sample standard deviation is $\sqrt{\frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)}$.]

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

5. Let n be a positive number.

(a) Expand $\frac{2}{e^{nx}}$ in ascending powers of x as far as the term in x^3 .

(b) Consider the expansion of $(1+4x)^m + \frac{2}{e^{nx}}$, where m is a positive integer. The coefficients of x and x^2 in the expansion are 24 and 980 respectively. Find the coefficient of x^3 in the expansion. (7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

- Answers written in the margins will not be marked.

8

Please stick the barcode label here.

7. A computer programme adjusts the length and the breadth of a rectangular digital picture, such that the length of its diagonal remains constant while its breadth decreases at the constant rate of 0.5 cm s^{-1} . Initially, the length and the breadth of the picture are 20 cm and 15 cm respectively. Denote the breadth of the picture by x cm. Find the rate of change of the area of the picture when $x = 7$. (4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.



(a) Using the Standard Normal Distribution Table on page 24, evaluate

$$\int_0^{0.5} \frac{-x^2}{e^2} dx$$

- (b) Consider the curve $C: y = (2x - 1)e^{\frac{-x^2}{2}}$, where $x \geq 0$. Using the result of (a), find the area of the region bounded by C , the x -axis and the y -axis.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

- Answers written in the margins will not be marked.

Weight of a pumpkin (W kg)	$W \leq 3.6$	$3.6 < W \leq 5.7$	$W > 5.7$
Grade	C	B	A
Price (\$)	50	80	100

Suppose that 8 pumpkins are randomly chosen in the market and these pumpkins are put into a trolley.

- (i) Find the expected price of the pumpkins in the trolley.
 - (ii) Find the probability that there are at least 5 grade B pumpkins and at least 1 grade A pumpkin in the trolley.
- (6 marks)

[illegible]

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

- Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

2024-DSE-MATH-EP(M1)-14

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.



Answers written in the margins will not be marked.

A large rectangular area with horizontal ruling lines, intended for writing answers.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

where t ($t \geq 0$) is the number of months elapsed since the shop opens.

- (b) Let P be the total profit (in thousand dollars) of the shop. It is given that

where t ($t \geq 0$) is the number of months elapsed since the shop opens.

- (9 marks)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

Answers written in the margins will not be marked.

2024-DSE-MATH-EP(M1)–20

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.



Answers written in the margins will not be marked.

A large rectangular area with horizontal ruling lines, intended for writing answers.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

END OF PAPER

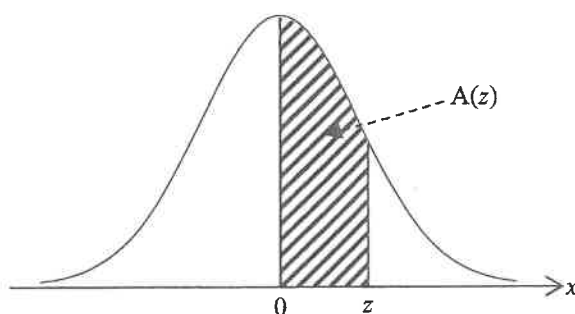
Answers written in the margins will not be marked.



Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ ($z \geq 0$). Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$